Statistical Moments Properties

Denote W, X, Y, and Z four random variables, and a, b, c, and d four arbitrary constants.

Expected value

$$E[X] = \begin{cases} \sum_i X_i p_i & \text{Discrete } X \\ \int_{-\infty}^{\infty} X f(X) dX & \text{Continuous } X \end{cases}$$

$$E[X+Y] = E[X] + E[Y]$$

$$E[aX] = aE[X]$$

$$E[a] = a$$

$$E[E[X]] = E[X]$$

$$E[XY] \neq E[X]E[Y] \quad \text{unless } X \perp Y$$

Variance

$$\text{Var}(X) = E[(X - E[X])^2]$$

$$\text{Var}(X) > 0$$

$$\text{Var}(a) = 0$$

$$\text{Var}(X+a) = \text{Var}(X)$$

$$\text{Var}(aX) = a^2\text{Var}(X)$$

$$\text{Var}(aX+bY) = a^2\text{Var}(X) + b^2\text{Var}(Y) + 2ab\text{Cov}(X,Y)$$

$$\text{Var}(aX-bY) = a^2\text{Var}(X) + b^2\text{Var}(Y) - 2ab\text{Cov}(X,Y)$$

Covariance

$$\text{Cov}(x,y) = E[(x - E[x])(y - E[y])]$$

$$\text{Cov}(X,a) = 0$$

$$\text{Cov}(X,X) = \text{Var}(X)$$

$$\text{Cov}(X,Y) = \text{Cov}(Y,X)$$

$$\text{Cov}(aX,bY) = ab\text{Cov}(X,Y)$$

$$\text{Cov}(X+a,Y+b) = \text{Cov}(X,Y)$$

$$\text{Cov}(aX+bY,cW+dZ) = ac\text{Cov}(X,W) + ad\text{Cov}(X,Z) + bc\text{Cov}(Y,W) + bd\text{Cov}(Y,Z)$$