Interpretation

Lecture 12

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Omitted variable bias

- If a third **variable** is correlated with both *x* and *y*, it would **bias the relationship**
 - We must then **control** for such variables
 - And if we can't we must acknowledge that our estimate is not causal with 'ceteris paribus'



Functional form

- Not capturing the **right functional** form correctly might also lead to biased estimations:
 - Polynomial order, interactions, logs, discretization matter
 - Visualizing the relationship is key



Selection bias

- Self-selection is also a common threat to causality
- What is the impact of going to a better neighborhood on your children outcomes?
 - We cannot just regress children outcomes on a mobility dummy
 - Individuals who move may be different from those who stay: **self-selection issue**
 - Here it is not that the sample is not representative of the population, but that the outcomes of those who stayed are different from the outcomes those who moved would have had, if they had stayed

Simultaneity

- Consider the relationship between crime rate and police coverage intensity
- What is the **direction of the relationship**?
 - We cannot just regress crime rate on police intensity
 - It's likely that more crime would cause a positive response in police activity
 - And also that police activity would deter crime

Measurement error

- Measurement error in the independent variable also induces a bias
 - The resulting estimation would mechanically be **downward biased**
 - The **noisier** the measure, the **larger the bias**



Randomized Controlled Trials

- A Randomized Controlled Trial (RCT) is a type of experiment in which the thing we want to know the impact of (called the treatment) is **randomly allocated** in the population
 - The two groups would then have the same characteristics on expectation, and would be comparable
 - It is a way to obtain **causality** from randomness
- RCTs are very **powerful tools** to sort out issues of:
 - Omitted variables
 - Selection bias
 - Simultaneity
- But RCTs are **not immune** to every problem:
 - The sample must be representative and large enough
 - Participants should comply with their treatment status
 - Independent variables must not be noisy measures of the variable of interest

o ...

Today: Interpretation

1. Point estimates

1.1. Continuous variables

1.2. Discrete variables

1.3. Log vs. level

2. Practice interpretation

3. Regression tables

3.1. Layout3.2. Reported significance3.3. R squared

4. Wrap up!

Today: Interpretation

1. Point estimates

- 1.1. Continuous variables
- 1.2. Discrete variables
- 1.3. Log vs. level

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1.1. Continuous variables

- In this first part, we're going to consider the **relationship** between:
 - The **income level** of young parents
 - The **health** of their **newborn**
- Consider first the following specification of the two variables:
 - A continuous measure of **annual household income in euros**
 - A continuous measure of **birth weight in grams**

 $\operatorname{Birth}\operatorname{weight}_i = \alpha + \beta \times \operatorname{Household}\operatorname{income}_i + \varepsilon_i$

lm(birth_weight ~ household_income, data)\$coefficients

- ## (Intercept) household_income
- ## 3.134528e+03 2.213871e-03

→ How would you interpret $\hat{\beta}$ here? (Note that e+03 and e-03 mean $\times 10^3$ and $\times 10^{-3}$)

1.1. Continuous variables

• When both x and y are continuous, the **general** template for the **interpretation** of $\hat{\beta}$ is:

"Everything else equal, a 1 [unit] increase in [x] is associated with an [in/de]crease of [beta] [units] in [y] on average."

• So in our case the **adequate interpretation** would be:

"Everything else equal, a **1 euro increase in annual household income** is associated with an **increase of 0.002 gram in newborn birth weight** on average."

- But it would be even better to **interpret** the results for a **meaningful variation** of x
 - For an annual household income, a **1 euro variation** is not really meaningful
 - 1 euro increase → 0.002 gram increase ⇔ 1,000 euro increase → 2 gram increase

1.1. Continuous variables

- A common way to obtain a coefficient for a **meaningful variation** of x is to **standardize** x
 - If we divide x by $\mathrm{SD}(x)$, the 1 **unit increase** in $\frac{x}{\mathrm{SD}(x)}$ is equivalent to an $\mathrm{SD}(x)$ **increase** in x
 - An $\mathrm{SD}(x)$ change in x is meaningful: it's low if x is very concentrated and high if x is highly spread out



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1.1. Continuous variables

Note that if you standardize both x and y, the resulting β̂ equals the correlation between x and y
 o To show that, let's first rewrite the formula of the beta coefficient:

$$\hat{eta} = rac{\mathrm{Cov}(x,\,y)}{\mathrm{Var}(x)} = rac{\mathrm{Cov}(x,\,y)}{\mathrm{SD}(x) imes\mathrm{SD}(x)}$$

$$\hat{eta} = rac{\mathrm{Cov}(x,\,y)}{\mathrm{SD}(x) imes\mathrm{SD}(x)} imesrac{\mathrm{SD}(y)}{\mathrm{SD}(y)}$$

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$$\hat{eta} = \operatorname{Cor}(x,\,y) imes rac{\operatorname{SD}(y)}{\operatorname{SD}(x)}$$

1.1. Continuous variables

• Starting with the previous expression, the $\hat{\beta}$ coefficient with the standardized variables writes:

$$\hat{eta} = rac{ ext{Cov}ig(rac{x}{ ext{SD}(x)}, rac{y}{ ext{SD}(y)}ig)}{ ext{SD}ig(rac{x}{ ext{SD}(x)}ig) imes ext{SD}ig(rac{y}{ ext{SD}(y)}ig)} imes rac{ ext{SD}ig(rac{y}{ ext{SD}(y)}ig)}{ ext{SD}ig(rac{x}{ ext{SD}(x)}ig)}$$

• But by construction, the standard deviation of a standardized variable is 1:

$$\hat{eta} = rac{ ext{Cov}ig(rac{x}{ ext{SD}(x)}, rac{y}{ ext{SD}(y)}ig)}{1 imes 1} imes rac{1}{1} \ \hat{eta} = ext{Cov}ig(rac{x}{ ext{SD}(x)}, rac{y}{ ext{SD}(y)}ig) \ \hat{eta} = rac{ ext{Cov}ig(x,y)}{ ext{SD}(x) imes ext{SD}(y)} = ext{Cor}(x,y)$$

• Learn the cheatsheet on moments properties:



1.2. Discrete variables

- Consider the following specification of the two variables:
 - A categorical variable for **annual household income divided in terciles**
 - Still a continuous measure of **birth weight in grams**

 $\mathrm{Birth}\ \mathrm{weight}_i = lpha + eta_1 \mathrm{T2}_i + eta_2 \mathrm{T3}_i + arepsilon_i$

• Recall that when including a categorical variable in a regression, a reference category must be omitted

lm(birth_weight ~ income_tercile, data)\$coefficients

##	(Intercept)	<pre>income_tercileT2</pre>	<pre>income_tercileT3</pre>
##	3112.83162	88.24778	222.65414

→ How would you interpret $\hat{\beta}_1$ and $\hat{\beta}_2$ here?

1.2. Discrete variables

• With a discrete *x*, the interpretation of the coefficient must be **relative to the reference category:**

"Everything else equal, belonging to the [x category] is associated with a [beta] [unit] [higher/lower] average [y] relative to the [reference category]."

• So in our case, the **adequate interpretations** would be:

"Everything else equal, belonging to the **second income tercile** is associated with a **88 grams higher average birth weight** relative to the **first income tercile**."

"Everything else equal, belonging to the **third income tercile** is associated with a **223 grams higher average birth weight** relative to the **first income tercile**."

• And the **intercept** is the **average birth weight** for newborns to parents in the **first income tercile**

1.2. Discrete variables

- Consider now the following specification of the two variables:
 - A continuous measure of **annual household income in euros**
 - A **binary variable** taking the value 1 if **the newborn is underweight** and 0 otherwise

Underweight_i = $\alpha + \beta \times$ Household income_i + ε_i

lm(underweight ~ household_income, data)\$coefficients

(Intercept) household_income
5.214013e-02 -4.084787e-07

 \rightarrow How would you interpret $\hat{\beta}$ here?

→ And would you consider its magnitude high?

1.2. Discrete variables

• With a **binary** *y* **variable**, the coefficient must be interpreted in **percentage points:**

"Everything else equal, a 1 [unit] increase in [x] is associated with a [beta \times 100] percentage point [in/de]crease in the probability that [y equals 1] on average."

• So in our case, the **adequate interpretation** would be:

"Everything else equal, a **1 euro increase in annual household income** is associated with a **0.00004 percentage point decrease in the probability that the newborn is underweight** on average."

- Here the interpretation would be more meaningful:
 - For a **1,000 euro** increase → 0.04 percentage point decrease
 - Compared to the **typical probability** to have an underweight newborn

1.2. Discrete variables

• The mean of a dummy variable corresponds to the share of 1s:

mean(data\$underweight)

[1] 0.037

• We can also compute the probability that y = 1 for the average x with our estimated coefficients:

5.214013e-02 + mean(data\$household_income) * -4.084787e-07

[1] 0.03700001

For the **average household**, a **1,000 euro increase** in annual income would be associated with a **0.0004 / 0.037 \approx 1\% decrease in the probability** that the newborn is **underweight**

1.2. Discrete variables

- Finally if **both** the *y* and the *x* variables are **discrete**, the coefficient must be interpreted:
 - In percentage points
 - **Relative** to the reference category

 $\mathrm{Underweight}_i = lpha + eta_1 \mathrm{T2}_i + eta_2 \mathrm{T3}_i + arepsilon_i$

lm(underweight ~ income_tercile, data)\$coefficients

(Intercept) income_tercileT2 income_tercileT3
0.07207207 -0.03903904 -0.06608405

"Everything else equal, belonging to the **second income tercile** is associated with a **3.9 percentage point lower probability that the newborn is underweight** relative to the **first income tercile.**"

1.3. Log vs. level

• Consider now the following hypothetical relationship:



- The slope tells us by how many **units** the *y* variable would increase for a **1 unit** increase in *x*
- But often times in Economics we're interested in the elasticity between the two variables:
 - What is the expected **percentage change** in *y* for a **one percent increase** in *x*?

→ The log transformation can be used to easily get an approximation of that

1.3. Log vs. level

• Instead of considering

 $y_i = lpha_{lvl} + eta_{lvl} x_i + arepsilon_i$



• We consider

 $\log(y_i) = lpha_{log} + eta_{log} \log(x_i) + arepsilon_i$



1.3. Log vs. level

 $\widehat{eta_{lvl}}=1.0121933$



 $(15\div100) imes\widehat{eta_{lvl}}pprox(15\div100) imes1.0121933\pprox0.151829$

 $\widehat{eta_{log}}=0.16875$



 $0.151829 \div 90 = 0.0016870 \ pprox eta_{log}\%$

1.3. Log vs. level

- Thus the interpretation differs depending on whether variables are in log or in level:
 - When variables are in **level** we should interpret the coefficients in terms of **unit** increase
 - When variables are in **log** we should interpret the coefficients in terms of **percentage** increase

	У	log(y)
x	\hat{eta} is the unit increase in y due to a 1 unit increase in x	$\hat{eta} imes 100$ is the % increase in y due to a 1 unit increase in x
log(x)	$\hat{eta} \div 100$ is the unit increase in y due to a 1% increase in x	\hat{eta} is the % increase in y due to a 1% increase in x

Interpretation of the regression coefficient

1.3. Log vs. level

- Let's give it a try with our example on household income and birth weight
 - We've already seen that because income is log-normally distributed, it should be included in log



1.3. Log vs. level

• So what would be your interpretation of the slope estimated from the following regression?

 $\mathrm{Birth}\ \mathrm{weight}_i = lpha + \beta \log(\mathrm{Household}\ \mathrm{income}_i) + \varepsilon$

lm(birth_weight ~ log(household_income), data)\$coefficients

##	(Intercept)	<pre>log(household_income)</pre>
##	2091.2323	112.3234

• With a continuous *y* in level and a logged *x* variable, the template would be:

"Everything else equal, a 1 percent increase in [x] is associated with a [beta/100] [unit] [in/de]crease in [y] on average."

• So in our case, the **adequate interpretation** would be:

"Everything else equal, a **1 percent increase in annual household income** is associated with a **1.12 grams increase in the birth weight of the newborn** on average."

Overview

1. Point estimates ✓

1.1. Continuous variables1.2. Discrete variables1.3. Log vs. level

2. Practice interpretation

3. Regression tables

3.1. Layout3.2. Reported significance3.3. R squared

4. Wrap up!

Overview

1. Point estimates ✓

1.1. Continuous variables1.2. Discrete variables

1.3. Log vs. level

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→ Let's practice coefficient interpration with randomly generated relationships:



Overview

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3.1. Layout

• So far we've been used to regression results displayed this way:

lm(birth_weight ~ household_income, data)\$coefficients

- ## (Intercept) household_income
 ## 3.134528e+03 2.213871e-03
 - Or with the more exhaustive **summary()** coefficients output:

summary(lm(birth_weight ~ household_income, data))\$coefficients

Estimate Std. Error t value Pr(>|t|)
(Intercept) 3.134528e+03 1.656840e+01 189.187165 0.000000e+00
household_income 2.213871e-03 2.808507e-04 7.882732 8.355367e-15

→ But in *formal* reports and academic papers, the *layout* of regression tables is *a bit different*

3.1. Layout

	Dependent variable:	
	Birth weight	
	(1)	(2)
Household income	0.002***	0.002***
	(0.0003)	(0.0003)
Girl (ref: Boy)		-135.218***
		(34.838)
Constant	3,134.528***	3,246.365***
	(16.568)	(34.257)
Observations	1,000	963
Note:	*p<0.1; **p<0.05; ***p<0.01	

Regression tables often contain multiple regressions:

- With one regression in each column
 - Regression models are numbered
 - Dependent variable mentioned above
- And one variable in **each row**
 - With the **point estimate**
 - And a **precision measure** below
- General info on each model at the bottom
- A **symbology** for the **p-value** testing whether the coefficient is significantly different from 0 or not

3.1. Layout

	Dependent variable:	
	Birth weight	
	(1)	(2)
Household income	0.002***	0.002***
	(0.0003)	(0.0003)
Girl (ref: Boy)		-135.218***
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Constant	3,134.528***	3,246.365***
	(16.568)	(34.257)
Observations	1,000	963
Note:	*p<0.1; **p<0.05; ***p<0.01	

It makes it easy to compare the different models:

We can add controls progressively

 Check the stability of the main coefficient

→ If it gets significantly closer to 0 it might indicate that the raw relationship was fallaciously driven by a confounding factor

- And compare general statistics
 - N is lower in the second regression
 - It means that there are missing values
 - Could this induce a selection bias?

3.2. Reported significance

	Dependent variable:	
	Birth weight	
	(1)	(2)
Household income	0.002***	0.002***
	(0.0003)	(0.0003)
Girl (ref: Boy)		-135.218***
		(34.838)
Constant	3,134.528 ^{***}	3,246.365***
	(16.568)	(34.257)
Observations	1,000	963
Note:	*p<0.1; **p<0.05; ***p<0.01	

It makes it easy to compare the different models:

- The evolution of the significance matters as well
 The main coefficient should stay significant
- But don't rely too much on the symbology
 - Thresholds are **not always the same**
 - Sometimes there are none
- Instead, keep in mind this **rule of thumb**:

→ A coefficient \approx twice larger than its standard error has a p-value of \approx 5%

3.2. Reported significance

- Remember the formula for the **confidence interval**:
 - $\circ~$ We can **fix** the **confidence level** 1-lpha to 95% and check **how** t **varies with** df



3.2. Reported significance

- As soon as you have about 20 observations more than you have parameters to estimate:
 - $\circ~$ The t value gets very close to 2
 - $\,\circ\,\,$ And as df increases it quickly converges to pprox 2
- The coefficient is statistically significant if the lower bound of its (absolute) confidence interval is larger than 0
 - $\circ~$ Which is an easy calculation if we **approximate the** t **value by 2**
 - A reasonable approximation for a back of the envelope calculation unless there are very few observations
- The *(absolute)* lower bound of the CI writes:

 $egin{aligned} &|\hat{eta}| - t(\mathrm{df})_{1-rac{lpha}{2}} imes \mathrm{se}(\hat{eta}) \ &|\hat{eta}| - 2 imes \mathrm{se}(\hat{eta}) > 0 \ &|\hat{eta}| > 2 imes \mathrm{se}(\hat{eta}) \end{aligned}$

So if the **coefficient** is clearly more than **twice larger** than it's **standard error**, it must be **statistically significant** at the **5% significance** level

→ But sometimes the p-value or the confidence interval is reported instead of the standard error

3.2. Reported significance

	Dependent variable:	
	Birth weight	
	(1)	(2)
Household income	0.002***	0.002***
	p = 0.000	p = 0.000
Girl (ref: Boy)		-135.218***
		p = 0.0002
Constant	3,134.528***	3,246.365***
	p = 0.000	p = 0.000
	1 000	062
Observations	1,000	963
Note:	*p<0.1; **p<0.05; ***p<0.01	

	Dependent variable:		
	Birth weight		
	(1)	(2)	
Household income	0.002***	0.002***	
	(0.002, 0.003)	(0.002, 0.003)	
Girl (ref: Boy)		-135.218***	
		(-203.500, -66.936)	
Constant	3,134.528***	3,246.365***	
	(3,102.055, 3,167.002)	(3,179.223, 3,313.507)	
Observations	1,000	963	
Note:	*p<0.1; **p<0.05; ***p<0.01		

3.3. R squared

- In **regression tables**, the **R**² of the model is **always reported** below the number of observations
 - $\circ~$ The R^2 captures how well the **model fits the data**

0

3.3. R squared

- In **regression tables**, the **R²** of the model is **always reported** below the number of observations
 - The R² captures how well the **model fits the data**
 - The model has a **good fit (high R²)** on dataset A but a **poor fit (low R²)** on dataset B



3.3. R squared

- The **standard error** already gives an idea on the goodness of the fit, but it is expressed in the **same unit as** y
 - So we **cannot compare** two different models based on that statistic

0

3.3. R squared

- The **standard error** already gives an idea on the goodness of the fit, but it is expressed in the **same unit as** *y*
 - So we **cannot compare** two different models based on that statistic
 - The standard error of the slope would be larger on dataset A than on dataset B



3.3. R squared

0

0

• The **R²** captures the **goodness of fit** as the **percentage** of the *y* variation captured by the model, from:



3.3. R squared

0

- The **R²** captures the **goodness of fit** as the **percentage** of the *y* variation captured by the model, from:
 - $\circ~$ The **total variation** of the y variable (its variance $\sum_{i=1}^n (y_i ar{y})^2$)



3.3. R squared

- The **R²** captures the **goodness of fit** as the **percentage** of the *y* variation captured by the model, from:
 - $\circ~$ The **total variation** of the y variable (its variance $\sum_{i=1}^n (y_i ar{y})^2$)
 - $\circ~$ The **remaining variation** of the y variable once its modeled (the sum of squared residuals $\sum_{i=1}^n \hat{arepsilon_i}^2$)



3.3. R squared

• We can then obtain a proper formula from the following reasoning

 $Total \ variation = Explained \ variation + Remaining \ variation$

 $rac{ ext{Explained variation}}{ ext{Total variation}} = 1 - rac{ ext{Remaining variation}}{ ext{Total variation}}$ $rac{ ext{Explained variation}}{ ext{Total variation}} = 1 - rac{ extsf{\sum}_{i=1}^{n} \hat{arepsilon}_{i}^{2}}{ extsf{\sum}_{i=1}^{n} (y_{i} - ar{y})^{2}} \equiv ext{R}^{2}$

- Because all the terms are sums of squares, we usually talk about:
 - Total Sum of Squares (TSS)
 - Explained Sum of Squares (ESS)
 - Residual Sum of Squares (RSS)

3.3. R squared

- Note that the **TSS** is actually the **variance of** *y*:
 - So the \mathbf{R}^2 is interpreted as the **share of the variance of** y which is **explained** by the model
 - $\circ~$ And as such, the R^2 is always comprised $between \ 0 \ and \ 1$

$$\mathrm{R}^2 = 1 - rac{{\sum_{i=1}^n {\hat{arepsilon_i}}^2}}{{\sum_{i=1}^n (y_i - ar{y})^2}} = rac{\mathrm{Explained\ variation}}{\mathrm{Total\ variation}}$$

- An undesirable property of the **R²** is that it **mechanically increases** with the number of **dependent variables**
 - Such that with many variables the R² tends to overestimate the goodness of the fit
 - This is why you will sometimes see some **Adjusted R²**

$$\mathrm{Adjusted} \ \mathrm{R}^2 = 1 - rac{(1-\mathrm{R}^2)(n-1)}{n-\#\mathrm{parameters}}$$

Overview

1. Point estimates ✓

1.1. Continuous variables1.2. Discrete variables1.3. Log vs. level

3. Regression tables \checkmark

3.1. Layout3.2. Reported significance3.3. R squared

2. Practice interpretation \checkmark

4. Wrap up!

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Standard interpretations

• When both x and y are continuous, the **general** template for the **interpretation** of \hat{eta} is:

"Everything else equal, a 1 [unit] increase in [x] is associated with an [in/de]crease of [beta] [units] in [y] on average."

• With a discrete *x*, the interpretation of the coefficient must be **relative to the reference category:**

"Everything else equal, belonging to the [x category] is associated with a [beta] [unit] [higher/lower] average [y] relative to the [reference category]."

• With a **binary** *y* **variable**, the coefficient must be interpreted in **percentage points:**

"Everything else equal, a 1 [unit] increase in [x] is associated with a [beta \times 100] percentage point [in/de]crease in the probability that [y equals 1] on average."

4. Wrap up!

Interpretations with variable transformation

Standardization

- To standardize a variable is to **divide it by its SD**
 - The variation of a standardized variable should not be **interpreted** in units but **in SD**
 - For instance if *x* and *y* are continuous and *x* is standardized, the interpretation becomes:

"Everything else equal, a 1 **standard deviation** increase in [x] is associated with an [in/de]crease of [beta] [units] in [y] on average."

• If both x and y are standardized, the slope is the correlation coefficient between x and y

Log-transformation

• The log transformation allows to interpret the coefficient in percentage:

Interpretation of the regression coefficient

	У	log(y)	
x	\hat{eta} is the unit increase in y due to a 1 unit increase in x	$\hat{eta} imes 100$ is the % increase in y due to a 1 unit increase in x	
log(x)	$\hat{eta} \div 100$ is the unit increase in y due to a 1% increase in x	\hat{eta} is the % increase in y due to a 1% increase in x	

4. Wrap up!

Regression table layout

	Birth weight	
	(1)	(2)
Household income	0.002***	0.002***
	(0.0003)	(0.0003)
Girl (ref: Boy)		-135.218***
		(34.838)
Constant	3,134.528***	3,246.365***
	(16.568)	(34.257)
Observations	1,000	963
R ²	0.059	0.074
Note:	*p<0.1; **p<0.05; ***p<0.01	

Regression tables often contain multiple regressions:

- With one regression in each column
- And one variable in each row
 With the point estimate
 - And a **precision measure** below
- General info on each model at the bottom • Number of observations • $R^2 = 1 - \frac{\sum_{i=1}^n \hat{\varepsilon_i}^2}{\sum_{i=1}^n (y_i - \bar{y})^2}$
- A **symbology** for the **p-value** testing whether the coefficient is significantly different from 0 or not